

THEORY OF RADIATION MECHANISMS OF PULSARS. I

H. Y. CHIU* AND V. CANUTO

Institute for Space Studies, Goddard Space Flight Center, National
 Aeronautics and Space Administration, New York, New York

Received 1970 January 15; revised 1970 May 22, and in proofs 1970 September 4

ABSTRACT

A possible mechanism for pulsar radiation is proposed. A maserlike action is shown to take place on account of a nonthermal distribution of the electrons due to the presence of an electric field. The radiation process takes place via Coulomb bremsstrahlung of the electrons in the presence of a strong quantizing magnetic field. The model predicts that the amplification of radiation in the radio range gives brightness temperatures ranging from 10^{21}° to over 10^{31}° K, depending on the individual pulsar. This is in the range of the observed brightness temperature of pulsars.

I. INTRODUCTION

It is now generally believed that pulsars are rotating magnetic neutron stars and that the observed (fundamental) period of the pulses is the period of rotation. The emission of pulsed radiation is the result of a collimated beam corotating with the star (Gold 1969). Observation shows that the radio emission takes place within a spatial region of dimension less than 30 km (Drake and Craft 1968). Simultaneous observations at X-ray and optical wavelengths show that pulses at these two bands are received within 10^{-3} sec (Bradt *et al.* 1969). The simplest interpretation to these coincidences in the arrival time is that *all* pulsar radiation originates from a common emission region less than 30 km in spatial extension, although the mechanism for optical and radio emission may be quite different.

Quantitative theories of pulsars have been advanced by Ostriker and Gunn (1969), Pacini (1968), and Goldreich and Julian (1969) to account for the slowdown rate of rotation and the production of cosmic-ray particles. However, no quantitative theory on the radiation mechanism is currently available.

The problem is to find a radiation mechanism such that the very large brightness temperature (ranging from 10^{24}° to 10^{31}° K for occasional short bursts) can be accounted for and, in addition, to explain the collimation of the emitted beam and the polarization of the pulses. In this paper we will present a quantitative theory to account for the high brightness temperature.

We will show that our theory can qualitatively account for the pulsed emission and the polarization. In this paper the optical or X-ray emission will not be discussed.

II. EMISSION REGION OF THE STAR

In a number of discussions the emissions of radio and optical radiation are assumed to take place at a distance away from the neutron star, and a preferred distance is one at which the velocity of rotation becomes the velocity of light (velocity-of-light circle). In this paper the emission region is taken to be at the surface of the neutron star. The reason for favoring surface emission over emission at a distance is as follows.

First, the velocity-of-light circle is at a distance of $4.8 \times 10^4 P$ km away from the star, where P is the period in seconds. In order to have pulsed emission, energetic particles must be transported to the velocity-of-light circle, either regularly at intervals

* Also with Physics Department and Earth and Space Sciences Department, State University of New York, Stony Brook, New York.

coincident with the pulsing rate or with such an anisotropic pattern that pulsed emission can take place. The difficulties involved in such a scheme of emission are many, and none of them have been overcome in any quantitative way. In fact, not even a basic formulation of the problem is available. In particular, the distance of the velocity-of-light circle depends linearly on the period, and this model would then predict a correlation between the period and the radio brightness. Observation shows that there is no such correlation. From the theoretical point of view, the extremely short lifetime of particles against synchrotron emission in an intense magnetic field makes this scheme almost unworkable. Further, it is hard to think of a mechanism by which the plasma is confined to a dimension of 30 km independent of the dimension of the size of the velocity-of-light circle. Second, recent discoveries of minute changes in the periods of some pulsars imply that the emission region must be very firmly attached to the star. The period of the Vela pulsar was found to suffer a "glitch" resulting in a decrease of the period by $10^{-6} P$ ($P = 0.089$ sec) and an increase in dP/dt by about 1 percent (Reichley and Downs 1969). Later it was found that the Crab Nebula pulsar NP 0532 also exhibits small sudden decreases in the period by about $10^{-9} P$ (Wilkinson 1969), together with a small temporary change in dP/dt . These changes in P and dP/dt have been interpreted to be due to minute changes in the internal structure of the star (Ruderman 1968, 1969). If we accept this interpretation, which is so far the most reasonable one offered, then the emission region must be very rigidly tied to the star. The simplest interpretation would be that the emission region is at the surface of the star. This view is the same as those of some geometrical models constructed earlier (Böhm-Vitense 1969; Radhakrishnan and Cooke 1969).

In the next section we will present some essential features of our model, and the rest of this paper will deal with details of our theory.

III. OUTLINES OF THE PRESENT MODEL

The particular model we have in mind is that of an oblique rotator; that is, the direction of the magnetic field of the star is inclined at an angle with respect to rotation. The field strength at the surface is of the order of 10^{12} gauss. This model is the same as that of Pacini (1968) and Ostriker and Gunn (1969).

In the corotating frame an electric field parallel to the magnetic field is present. This field originates from the rotation of the star, as suggested by Goldreich and Julian (1969). The field strength is of the order of 10^{11} V cm $^{-1}$ for a pulsar rotating with a period of 1 sec.

This electric field causes particles of one type of charge to be accelerated away from the surface. A surface electric current along the magnetic field exists, and the effect of electrons at the surface therefore possesses a coherent streaming motion with a macroscopic velocity v_m (the macroscopic momentum is p_m). The distribution function of the electron, $f(p)$, in one idealized model can be approximated by that of a displaced Maxwellian distribution:

$$f(p) = C \exp \left[-\frac{(p - p_m)^2}{2mkT} \right], \quad C = \text{normalization constant}, \quad (1)$$

where p is the electron momentum.

This coherent motion gives rise to a negative temperature in parts of the electron spectrum where $p < p_m$. (A negative temperature is understood to be present when $df(p)/dp > 0$.) As will be shown later, if the transition probability $W(p)$ does not increase faster than the first power of p , i.e.,

$$\frac{d}{dp} [W(p)/p] < 0, \quad (2)$$

the absorption coefficient of radiation (including stimulated emission) can become negative so that amplification of radiation via stimulated emission (maser) is possible. This maser differs from the ordinary one in that it can amplify radiation over a continuum. Hereafter this continuum maser will be referred to as a C-maser.

Electrons in fields of the order of 10^{12} gauss are essentially one-dimensional particles, and the transition probability for bremsstrahlung in the ground-state Landau level has a dependence on p different from that for the three-dimensional case. We have shown that in the case of strong magnetic fields (Canuto and Chiu 1970)

$$W(p) \propto p^{-1}. \quad (3)$$

Hence equation (2) is satisfied, and an electron gas in a strong field becomes a C-maser if an electric current is introduced. The frequency response of this C-maser amplifier is a flat one; thus, radiation can be amplified over a wide frequency band.

The propagation of the emitted radiation is strongly affected by the imbedding plasma and the magnetic field. At $H \sim 10^{12}$ gauss, neutral plasma with particle density less than 10^{28} cm^{-3} is transparent to all frequencies up to the ion Larmor frequency, which is of the order of 10^{16} Hz . As the presence of a magnetic field and particle motion is important in causing the C-maser effect, only the pole regions can emit radiation. A conical beam results from this anisotropic emission, and the rotation of the star sweeps the radiation through space, causing an observer to detect pulses as the beam sweeps by in beaconlike fashion (Fig. 1).

IV. EMISSION OF ELECTROMAGNETIC WAVES IN A STRONG MAGNETIC FIELD

a) *One-Dimensional Properties of the Electron*

In a magnetic field the trajectories of the electrons are helices. In intense magnetic fields, when the radius of the helix is comparable to the de Broglie wavelengths of electrons, a quantization process similar to that for atomic electrons then applies, resulting in discrete energy levels (known as Landau levels) in the motion perpendicular to the magnetic field, and with the motion along the direction of the magnetic field left unchanged. The expression for the energy of the electron is then (Canuto and Chiu 1968)

$$E/mc^2 = [1 + (p_z/mc)^2 + 2nH/H_q]^{1/2} \quad (4)$$

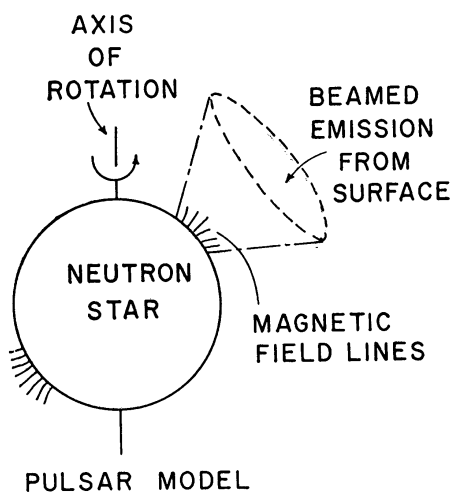


FIG. 1.—Configuration of emission. Note that the beaming is highly exaggerated here. The actual beaming is less pronounced and is discussed in § X.

where $H_q = m^2 c^3 / e \hbar = 4.414 \times 10^{13}$ gauss and $n = 0, 1, 2, \dots$, is the quantum number characterizing the size of the electron orbits. As a comparison with the usual expression for energy,

$$E/mc^2 = [1 + (p_z/mc)^2 + (p_x/mc)^2 + (p_y/mc)^2]^{1/2}, \quad (5)$$

we see that the quantization of orbits replaces

$$(p_x^2 + p_y^2)/m^2 c^2 \quad \text{by} \quad 2nH/H_q. \quad (6)$$

Consider an electron of energy less than $mc^2(1 + 2H/H_q)^{1/2}$. This electron can only stay in the lowest orbital state $n = 0$, such that its energy is simply

$$E/mc^2 = [1 + (p_z/mc)^2]^{1/2}, \quad (7)$$

which in the nonrelativistic case becomes

$$E = p_z^2/2m. \quad (8)$$

This is the expression of the energy for a one-dimensional particle.

This one-dimensional behavior is mandatory for electrons of energy less than the level spacing between the $n = 1$ and the $n = 0$ Landau level. Even when a few of the higher energy levels are available to the electrons, this one-dimensional behavior is still retained. Electrons in each Landau level can be regarded as a new particle state of a one-dimensional particle whose mass is $m_n = m(1 + 2nH/H_q)^{1/2}$ and whose kinetic energy (nonrelativistic expression) is just

$$E_n = p_z^2/2m_n. \quad (9)$$

This one-dimensional character, however, is gradually lost as more states become available (i.e., when the energy of the electron is large compared with the level spacing).

b) Radiation Processes

In a strong field the Zeeman splitting of atomic levels becomes large compared with the energy level itself. It is not known if any bound state can exist in a field of 10^{12} gauss. We will assume that, at the surface of a neutron star, atoms will be completely stripped of their atomic electrons. Hence the only radiating electron we will consider are electrons in Landau levels.

We will consider a path length of a few centimeters beneath the surface of the neutron star. In this case the only two important radiation processes are:

1. "Synchrotron" radiation, or a transition between two Landau levels. This is a spontaneous transition whereby an electron jumps from a quantum orbit n to another orbit n' ($n > n'$):

$$e^-(n) \rightarrow e^-(n') + \gamma. \quad (10)$$

The energy of the photon is dependent on the angle of emission, but in the nonrelativistic case and $H \ll H_q$ it is roughly $(n - n') (H/H_q) mc^2$. This expression for the energy of the photon is also applicable to the relativistic case if the emission is perpendicular to the field. This process is the quantized version of the classical synchrotron radiation.

2. Bremsstrahlung radiation:

$$e^-(n) + (Z, A) \rightarrow e^-(n') + (Z, A) + \gamma. \quad (11)$$

The spectrum of the emitted photon is a continuum up to the electron energy.

For process 1 above, the discrete transition between the Landau levels gives rise to radiation similar to atomic line transitions, but the "atomic" level here is an ill-defined

one because the energy of the photon depends on the angle of emission as well as the strength of the field. For weak fields and relativistic electron energy, the level spacing is

$$\Delta E = \frac{H/H_q}{E/mc^2} mc^2; \quad (12)$$

and the frequency corresponding to this transition is

$$\nu_c = \Delta E/\hbar = 10^7 H \gamma^{-1} \text{ Hz}, \quad \gamma = E/mc^2, \quad (13)$$

where ν_c is also the minimum energy of emission. For the Crab Nebula, $H \sim 10^{-3}$, $\gamma = 10^3$, we find that $\nu_c \sim 10$ Hz. The spectrum of the resultant radiation from transitions among different Landau levels is essentially a continuum. This type of radiation is referred to as "synchrotron radiation" since it was first found to be exceedingly important in the design of electron synchrotrons. The frequency ν_c increases with the field strength. At $H \sim 10^{12}$ gauss, electrons of the same energy as those in the Crab Nebula will give rise to radiation with a level spacing of $\Delta E = 10$ eV, or $\nu_c \simeq 10^{16}$ Hz, well in the ultraviolet regime. In order to have radio emission at $\nu = 10^8$ Hz, the energy of the electron must exceed 10^{16} eV. The lifetime of such an electron in a field of 10^{12} gauss is much less than 10^{-19} sec.

The transition probability for discrete transitions as well as for the classical limit has been extensively discussed in the literature (Erber 1966; Chiu and Fassio-Canuto 1969).

The process of bremsstrahlung radiation has been calculated by Goldman and Oster (1964), but their result is very difficult to apply. We have recently calculated the rate of nonrelativistic bremsstrahlung radiation in a magnetic field (Canuto and Chiu 1970). In this calculation the electron wave function in a magnetic field is used in conjunction with a Green's function appropriate for the field. This calculation is therefore expected to be valid for nonrelativistic electrons with energies up to, say, 100 or 200 keV, in fields up to a fraction of $H_q = 4.414 \times 10^{13}$ gauss, for frequencies not in the vicinity of the plasma frequency or the Larmor frequency of the electrons or ions.

The expression for the transition probability at forward angle is (Canuto and Chiu 1970)

$$W = W_0 C_1(\lambda) (p\omega)^{-1} (\text{cm}^3 \text{ sec}^{-1}), \quad (14)$$

where ($\epsilon = E/mc^2$)

$$W_0 = Z^2 \alpha^3 \pi^2 N_i \chi_e^3 \hbar c^2 (H/H_q)^{-2} = 2.09 \times 10^{-37} Z^2 N_i (H/H_q) \quad (\text{erg cm}^2 \text{ sec}^{-1}),$$

$$N_i = \text{ion density}, \quad (15)$$

$$C_1(\lambda) = e^\lambda (1 + \lambda) E(\lambda) - 1, \quad \lambda \simeq \frac{\chi_e^2}{2\omega_H} [(\omega/v)^2 + D^{-2}],$$

and

$$E(\lambda) = \int_\lambda^\infty x^{-1} \exp(-x) dx (\lambda \rightarrow 0) \rightarrow \ln(\gamma\lambda)^{-1}, \quad \gamma = 1.78102 \dots \quad (16)$$

Here D is the screening distance of the scatterer. (In the presence of a magnetic field the theory of screening is different from that without it. Here we tacitly assume that the Debye theory applies.) It is seen that for $\omega \simeq 10^8 \text{ sec}^{-1}$, $v \simeq 10^9 \text{ cm sec}^{-1}$, $\omega/v \simeq 10^{-1} \text{ cm}$, and therefore $\lambda \simeq \omega^2/v^2$ only for impact parameter greater than ~ 10 cm. For smaller cutoff distances $D^{-2} \gg \omega^2/v^2$ and therefore λ does not depend on ω^2 , being merely a constant.

In the regime of interest $\lambda \ll 1$ we can write

$$W = W_0 (p\omega)^{-1} \ln(\gamma\lambda)^{-1} \quad (17)$$

The effect of a dense plasma is to alter the relation between the photon frequency ω and the wave vector κ of the photon. As will be discussed in § X, equation (14), though given in the vacuum limit, is still valid in a dense plasma $N_e \simeq 10^{24} \text{ cm}^{-3}$ embedded in a strong magnetic field $\sim 10^{12}$ gauss at forward direction ($\theta = 0$) for both modes of propagation (the ordinary and extraordinary modes).

V. BOLTZMANN EQUATION OF TRANSPORT

Generally speaking, the Boltzmann equation, which deals with nonequilibrium configurations, cannot be solved easily when an electric field and a magnetic field are simultaneously present. In the case which we are considering now, the electron orbits are strongly quantized, and the transition between magnetic states is regarded as an exchange of particles among these particle states. The effect of the magnetic field on particle motions is thus taken into account if we write one Boltzmann equation for each particle state and if we include in the collision term the transition probabilities for going into and out of other particle states. In particular, we will consider a special case, that all inhomogeneities are along the magnetic-field lines and all particles are in the ground state. This corresponds to the magnetic pole region of a neutron star with a field of $\sim 10^{12}$ gauss and an electron energy $\sim \text{keV}$.

In the absence of density and temperature inhomogeneities, the equation for the electron is

$$\frac{\partial}{\partial t} f(\mathbf{p}) + eE \frac{\partial}{\partial p} f(\mathbf{p}) + \frac{e}{c} \mathbf{p} \times \mathbf{H} \frac{\partial}{\partial \mathbf{p}} f(\mathbf{p}) = \left[\frac{d}{dt} f(\mathbf{p}) \right]_c; \quad (18)$$

and that for the photon is

$$\frac{\partial}{\partial t} F(\omega) + c\boldsymbol{\Omega} \cdot \frac{\partial}{\partial \mathbf{s}} F(\omega) = \left[\frac{d}{dt} F(\omega) \right]_c. \quad (19)$$

The origin of these equations are thoroughly discussed in literature, and we will not go into details (Chiu 1968). The functions $f(\mathbf{p})$ and $F(\omega)$ are electron and photon occupation numbers per state, E is the electric field in the z -direction, and \mathbf{p} is the electron z -momentum (the other two components have been replaced by the quantized expression); ω is the photon frequency, and $\boldsymbol{\Omega}$ is the direction vector of the photon. The terms $[(d/dt)f(\mathbf{p})]_c$ and $[(d/dt)F(\omega)]_c$ are "collision" terms describing the rate of change of the occupation number per state due to interactions. In our case $\mathbf{p} \parallel \mathbf{H}$, hence the $\mathbf{p} \times \mathbf{H}$ term drops out.

We will first consider the electron-collision term. This collision term consists of two parts, one due to elastic scattering by the nucleus through its Coulomb field, and the other due to bremsstrahlung photon emission. (We hereby neglect the discrete level transitions.) The elastic scattering is quite different from the usual elastic scattering process in that there is only one component of the momentum. For example, in an ordinary plasma electron-electron scattering is the chief mechanism by which the Maxwellian distribution is restored. In our case it is easily shown that the electron-electron scattering will result only in the exchange of momenta of the two particles, and is therefore equivalent to no collision at all. In the electron-ion collision the energy exchange is of the order of m/M_i , where M_i is the mass of the ion. The restoration of Maxwellian distribution will therefore take many more collisions than in the case of an ordinary plasma.

To the first approximation the electron energy can be regarded as a conserved quantity in electron-ion collisions. The relation between the initial and the final electron momenta \mathbf{p} and \mathbf{p}' is

$$p^2/2m = p'^2/2m \quad \text{or} \quad p' = \pm p. \quad (20)$$

The case $p' = p$ corresponds to forward scattering and is equivalent to no scattering taking place at all. The case $p' = -p$ corresponds to backward scattering and is the only one that should be taken into account.¹

The number of particles leaving the state p to the state $-p$ is

$$dN^- = (g/\hbar)f(p)d p W(p \rightarrow -p), \quad (21)$$

where $W(p \rightarrow -p)$ is the transition probability of $p \rightarrow -p$ per unit time per electron for an ion density N , and g is the statistical weight of the electron state in the ground Landau level and is (Chiu and Canuto 1970)

$$g = V(2\pi)^{-2}(mc^2/\hbar)^2(H/H_q), \quad V = \text{volume}, \quad (g/\hbar) \int_{-\infty}^{+\infty} f(p)dp = N_e. \quad (22)$$

The number of particles leaving the state $-p$ to go to the state p is

$$dN^+ = (g/\hbar)f(-p)d p W_e(-p \rightarrow p). \quad (23)$$

Therefore, the collision term for Coulomb scattering is

$$\left[\frac{d}{dt} f(p) \right]_{c.s.} = \frac{dN^+ - dN^-}{(g/\hbar)dp} = -f(p)W_c(p \rightarrow -p) + f(-p)W_c(-p \rightarrow p). \quad (24)$$

The collision term for the emission and absorption of photons is more complicated. An electron can make transitions from the state p to p' and p'' with the emission or absorption of a photon such that

$$p^2/2m + \hbar\omega = p'^2/2m, \quad (25)$$

$$p^2/2m = p'^2/2m + \hbar\omega. \quad (26)$$

These transitions and the corresponding transition probabilities are defined in Figure 2. Note that the transition probability here has the unit of transitions per unit time per electron per photon state per unit volume for an ion density N_i ; a slash denotes absorption processes.

The transition probabilities for emission and absorption are related by the detailed balancing theorem, which for a one-dimensional gas (whose phase-space element is

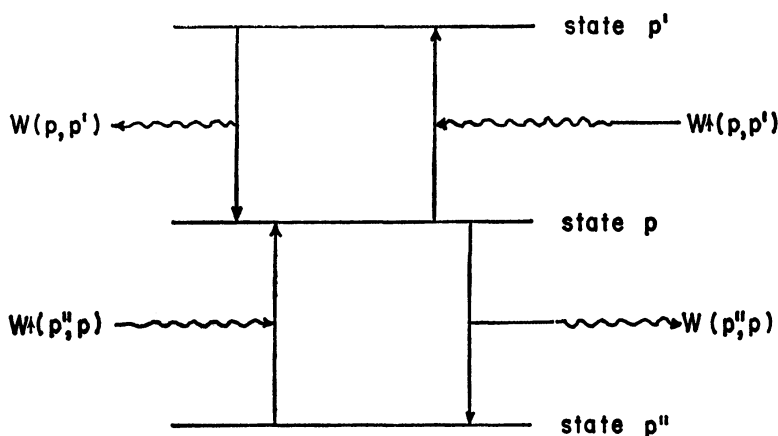


FIG. 2 —Definitions of the transition probabilities. Upward transitions are indicated by an arrow.

¹ Most recent work on intense magnetic fields has been summarized in an article by Chiu and Canuto (1970).

simply $(g/h)dp$, where g is the statistical weight) is simply

$$gW^\uparrow(p, p')dp = gW(p, p')dp', \quad gW^\uparrow(p'', p)dp'' = gW(p'', p)dp. \quad (27)$$

From equations (25) and (26) we find

$$\frac{dp'}{dp} = \frac{p}{p'}, \quad \frac{dp}{dp''} = \frac{p''}{p}. \quad (28)$$

The derivation of $[(d/dt)f]_c$ for photon emission and absorption is similar to that for equation (24). For the upward-transition case, equation (25) (denoted by the superscript A), we find

$$\begin{aligned} \left[\frac{d}{dt} f \right]^{(A)} &= \int g_\gamma h^{-3} d^3 p_\gamma F(\omega) [f(p') - f(p)] W^\uparrow(p, p') \\ &\quad - \int g_\gamma h^{-3} d^3 p_\gamma f(p') W^\uparrow(p, p'); \end{aligned} \quad (29)$$

and for the downward transition, equation (26) (denoted by the superscript B), we have

$$\begin{aligned} \left[\frac{d}{dt} f \right]^{(B)} &= \int g_\gamma h^{-3} d^3 p_\gamma F(\omega) [f(p'') - f(p)] W(p'', p) \\ &\quad - \int g_\gamma h^{-3} d^3 p_\gamma f(p) W(p'', p), \end{aligned} \quad (30)$$

with the understanding that in equation (30) the transition probability must not become imaginary (i.e., $\hbar\omega < p^2/2m$). The quantity g_γ is the statistical weight of the photon, and p_γ is the photon momentum.

The photon-collision term is similarly derived. The result is

$$\begin{aligned} \left[\frac{d}{dt} F(\omega) \right]_c &= \int_{-\infty}^{+\infty} g h^{-1} dp f(p) W^\uparrow(p, p') \\ &\quad + F(\omega) \int_{-\infty}^{+\infty} g h^{-1} [f(p') - f(p)] W^\uparrow(p, p') dp. \end{aligned} \quad (31)$$

The first term on the right-hand side of equation (31) gives spontaneous emission; the second term gives induced emission $f(p')$ and absorption $-f(p)$.

We are interested in the following case: The electron energy is of the order of keV, and the frequency of the emitted photon is in the radio regime. In this case, $\hbar\omega \ll p^2/2m$. Since

$$p' - p = \frac{2m\hbar\omega}{p + p'} \ll p, \quad (32)$$

we can write

$$f(p') - f(p) = \frac{\partial f}{\partial p} \Delta p, \quad \Delta p = p' - p. \quad (33)$$

Therefore,

$$\left[\frac{d}{dt} F(\omega) \right]_c = \int_{-\infty}^{+\infty} g h^{-1} dp f(p) W^\uparrow(p, p') + F(\omega) \int_{-\infty}^{+\infty} g h^{-1} dp \left[\frac{\partial f}{\partial p} \Delta p \right] W^\uparrow(p, p'). \quad (34)$$

Partially integrating the second term, since $W^\uparrow(p, p') p^{-1} f(p) = 0$ at $p = \pm \infty$, we have:

$$\begin{aligned} \left[\frac{d}{dt} F(\omega) \right]_c &= \int_{-\infty}^{+\infty} g h^{-1} dp f(p) W^\uparrow(p, p') \\ &\quad - 2m\hbar\omega F(\omega) \int_0^\infty g h^{-1} dp \left[f(p) \frac{\partial}{\partial p} \frac{W^\uparrow(p, p')}{p + p'} + f(-p) \frac{\partial}{\partial p} \frac{W^\uparrow(p, p')}{p' - p} \right]. \end{aligned} \quad (35)$$

The equation of radiative transfer is therefore

$$\frac{\partial}{\partial t} F(\omega) + c\Omega \frac{\partial}{\partial s} F(\omega) = \int_{-\infty}^{+\infty} f(p) gh^{-1} W^{\uparrow}(p, p') \\ - F(\omega) 2m\hbar\omega \int_0^{\infty} gh^{-1} dp \left[f(p) \frac{\partial}{\partial p} \frac{W^{\uparrow}(p, p')}{p + p'} + f(-p) \frac{\partial}{\partial p} \frac{W^{\uparrow}(p, p')}{p' - p} \right]. \quad (36)$$

In a steady state $(d/dt)f(\omega) = 0$; and if we neglect the spontaneous-emission term (which only gives noise in the theory of lasers), the equation of radiative transfer possesses the following solution:

$$F(\omega, s) = F(\omega, 0) \exp \left[- \int_0^s a(\omega) ds \right], \quad (37)$$

where

$$a(\omega) = 2m\hbar\omega \int_0^{\infty} gh^{-1} dp \left[f(p) \frac{\partial}{\partial p} \frac{W^{\uparrow}(p, p')}{p + p'} + f(-p) \frac{\partial}{\partial p} \frac{W^{\uparrow}(p, p')}{p' - p} \right]. \quad (38)$$

Amplification of radiation (maser effect) is possible if $a(\omega) < 0$. The condition for this is not straightforward, as is clear from the form of equation (14). However, if the electron distribution function is asymmetrical, e.g., $f(p) \gg f(-p)$ (this corresponds to a population inversion in terms of a coherent streaming motion in the $+p$ direction with respect to the medium, as discussed in § III), then the condition for amplification of radiation is just

$$\frac{\partial}{\partial p} \frac{W^{\uparrow}(p, p')}{p + p'} < 0, \quad (39)$$

or $W(p, p')$ must not rise faster than the first power of p . As is clear from the form of W for the $0 \rightarrow 0$ transition, equation (14), this condition is completely fulfilled. Hence it is possible to have a negative emission coefficient. A treatment by Simon and Strange (1969) giving the criterion

$$\frac{\partial}{\partial p} W(p) < 0 \quad (40)$$

has been found in error (Chiu and Canuto 1969a). The correct treatment is the one given above.

VI. SOLUTION OF THE BOLTZMANN EQUATION, C-MASER

We now look for solutions of the Boltzmann equations (18) and (19) with the collision term given by equations (29), (30), and (31). First, as we will show later in § VII, Coulomb scattering is relatively unimportant, and will be neglected accordingly. Second, we are primarily interested in steady-state solutions, so that we will set $(\partial/\partial t)f(p)$ and $(\partial/\partial t)F(\omega)$ equal to zero separately. Instead of solving for the electron distribution function, we will first assume that the displaced Maxwellian distribution function, equation (1), can be used. Once the electron distribution function is specified, we can look for self-consistent solutions.

The standard procedure of solving the Boltzmann equation is used. First we multiply equation (18) by $(g/\hbar)p$ and integrate over dp . By partial integration the left-hand side of equation (18) becomes

$$\int_{-\infty}^{\infty} eE \frac{\partial f}{\partial p} gh^{-1} dp = eEgh^{-1} \int_{-\infty}^{+\infty} p df = -eEgh^{-1} \int_{-\infty}^{+\infty} f dp = -eEN_e, \quad (41)$$

where N_e is the electron density. The same procedure applied to $[(d/dt)f(p)]^{(A,B)}$ yields:

$$-eEN_e = \int g_\gamma h^{-3} F(\omega) d^3 p_\gamma \int_{-\infty}^{+\infty} g h^{-1} dp \{ [f(p') - f(p)] W^\uparrow(p, p') \\ + [f(p'') - f(p)] W(p'', p) \}. \quad (42)$$

If we approximate $f(p') - f(p)$ and $f(p'') - f(p)$ by differentials, as in the case of $[(d/dt)F(\omega)]_e$, equation (42) becomes:

$$-eEN_e = \int F(\omega) g_\gamma h^{-3} d^3 p_\gamma \int_{-\infty}^{+\infty} g h^{-1} p dp [(p' - p) W^\uparrow(p, p') \\ + (p'' - p) W(p'', p)] \frac{\partial f(p)}{\partial p} \\ + \frac{1}{2} \int F(\omega) g_\gamma h^{-3} d^3 p_\gamma \int_{-\infty}^{+\infty} g h^{-1} p dp [(p' - p)^2 \frac{p}{p'} W(p, p') \\ + (p - p'')^2 W(p'', p)] \frac{\partial^2 f(p)}{\partial p^2}. \quad (43)$$

First we use the detailed balancing theorem (27) to relate W^\uparrow to W ; next we use the definition of $W(p', p)$ as given by:

$$W(p', p) = W_0(p\omega)^{-1} \ln(\gamma\lambda)^{-1}. \quad (44)$$

Expanding p' and p'' in terms of p , we obtain

$$eEN_e = \frac{3}{4} \int F(\omega) g_\gamma h^{-3} d^3 p_\gamma \int_{-\infty}^{+\infty} p dp \frac{\partial f(p)}{\partial p} \frac{(2m\hbar\omega)^2}{p^4} \\ - \frac{1}{4} \int F(\omega) g_\gamma h^{-3} d^3 p_\gamma \int_{-\infty}^{+\infty} p dp \frac{\partial^2 f(p)}{\partial p^2} \frac{(2m\hbar\omega)^2}{p^4}. \quad (45)$$

The equation of radiative transfer (36) can now be solved in conjunction with equation (45). Since $f(p)$ appears only under the integral, it is possible to assume a certain form of $f(p)$ and then to search for self-consistent solutions. To see the physical essence of the solution, let us assume $f(p)$ to be a δ -function at p_m :

$$f(p) = N^* \delta(p - p_m), \quad N^* = hg^{-1}N_e, \quad (46)$$

where N^* is a normalization constant. This type of distribution function is that of a zero-temperature Maxwellian gas displaced by the momentum p_m . As we will see in § X, this approximation does not lead to any gross error.

After integration, N_e cancels on both sides of equation (45); and the resultant equation is

$$eE = g_\gamma h^{-3} \int d^3 p_\gamma F(\omega) (2m)^2 \frac{5}{4} W_0 \ln(\gamma\lambda)^{-1} (\hbar\omega)^2 p_m^{-4} \omega^{-1} \quad (47)$$

$$= \frac{5}{4} (2m)^2 \hbar W_0 p_m^{-4} g_\gamma h^{-3} \ln(\gamma\lambda)^{-1} \int d^3 p_\gamma \hbar\omega F(\omega) \quad (48)$$

$$= \frac{5}{4} (2m)^2 \hbar W_0 p_m^{-4} E_\gamma \ln(\gamma\lambda)^{-1}, \quad (49)$$

where $E_\gamma = (g_\gamma/h^3)d^3pF(\omega)\hbar\omega$ is the radiation-energy density. The same treatment for the photon equation yields the following absorption coefficient:

$$\alpha(\omega) = -2m\hbar c^{-1}N_eW_0p_m^{-3}\ln(\gamma\lambda)^{-1}. \quad (50)$$

The absorption coefficient is therefore negative, and amplification of radiation takes place. Apart from the slowly varying factor $\ln(\gamma\lambda)^{-1}$, the amplification is frequency-independent. This means that our “amplifier” has a broad-band frequency response.

Multiplying equation (36) by $2h(2\pi\omega)^3/c^2$ to convert $F(\omega)$ to the intensity $I(\omega, s)$, we obtain

$$\frac{d}{ds}I(\omega, s) = -I(\omega, s)\alpha(\omega). \quad (51)$$

As before, we neglect spontaneous emission as it gives rise only to incoherent noise. The solution of equation (51) is

$$I(\omega, s) = I(\omega, 0) \exp \left[\int_0^s p_m^{-3} \delta \ln(\gamma\lambda)^{-1} ds \right], \quad (52)$$

where

$$\delta = 2\pi^2(N_e\chi_e^3)(N_i\chi_e^3)Z^2\alpha^3(H/H_q)^{-2}\chi_e^{-1}(mc)^3 \quad (53)$$

and s is the path length over which amplification takes place.

Before we proceed further, we will discuss a very important effect in the theory of light and microwave amplification by stimulated emission. This is the effect of saturation of amplification. As it turns out, the amplification process is so efficient that the output becomes saturated under virtually all parameters believed to be applicable to magnetic neutron stars.

VII. SATURATION OF STIMULATED EMISSION

If a small signal is fed into an amplifier (such as an audio amplifier), the output power is proportional to the input signal. However, when the output power reaches the inherent power limit of the amplifier, further increase in the input signal level will not increase the output power. When this happens, the amplifier no longer amplifies the input signal linearly, and this amplifier is said to have reached saturation.

The same saturation effect also exists in amplification by stimulated emission.² Let us consider a two-level system. The reasoning can be easily extended to continuum energy-level systems. Let there be n_2 particles in the upper energy state 2, and let n_1 be the number of particles in the lower state 1. In the absence of stimulated radiation, the lifetime of the state 2 is τ_2 . Let $N = n_2 - n_1$ be the difference between particles in the upper and the lower state, and population inversion is achieved when $N > 0$. Let N_0 be the steady-state value of N (N_0 depends on the pumping rate and other properties of the system). The rate equation for N in the absence of stimulated radiation is

$$\frac{dN}{dt} = \frac{N_0 - N}{\tau_2}. \quad (54)$$

Now let us introduce stimulated radiation and let the transition probability via stimulated emission be P_{21} . The rate of change of N due to stimulated emission is simply $-2NP_{21}$. (The factor 2 arises from the fact that each transition from the state 2 to the state 1 changes the value of N by 2.) The rate equation (54) now becomes

$$\frac{dN}{dt} = \frac{N_0 - N}{\tau_2} - 2NP_{21}. \quad (55)$$

² This is very extensively discussed in the literature; see, for example, Troup (1963).

A steady state obtains when N_s satisfies

$$\frac{N_0 - N_s}{\tau_2} - 2N_s P_{21} = 0 \quad (56)$$

or when

$$\frac{N_s}{N_0} = \frac{1}{1 + 2P_{21}\tau_2}. \quad (57)$$

As P_{21} is proportional to the intensity of stimulated radiation, $I(\omega, s)$, when $I(\omega, s)$ becomes large such that $2P_{21}\tau_2 \sim 1$, the value of N_s begins to decrease appreciably from the original value N_0 , and this laser amplifier no longer amplifies linearly; the “gain” decreases with increased output. Further increase in $I(\omega, s)$ will only cause a further decrease in the gain; and finally, as N_s approaches zero, the absorption coefficient, being proportional to N , also becomes zero and the medium becomes transparent to radiation. The medium no longer amplifies. The coherent radiation has reached its maximum intensity. This is the saturation effect in stimulated emission.

We can obtain the saturated intensity from the condition for the onset of saturation:

$$2P_{21}\tau_2 \approx 1, \quad (58)$$

where P_{21} is the transition probability for emission per photon state. It is easily seen from the Boltzmann equation that the photon distribution function is just

$$P_{21} = F(\omega)\alpha(\omega), \quad (59)$$

where τ_2^{-1} is the transition probability for spontaneous emission, or collision, whichever is greater. The collision rate is about 10^{10} per second. As we will see, the transition probability for spontaneous emission is much greater. The expression for τ_2^{-1} for spontaneous emission is easily seen from the Boltzmann equation (36) to be

$$\tau_2^{-1} = N_e W(p, p') = 2.09 \times 10^{-37} Z^2 N_e N_i (p\omega)^{-1} (H/H_q)^{-2} c_1(\lambda). \quad (60)$$

Numerically, if we set $N_e = 10^{24} \text{ cm}^{-3}$, $N_i = N_e/26$ (ion composition at the surface), $p = p_m = 0.01 \text{ } mc$ (see § VIII), $H/H_q = 0.02$ ($H \simeq 10^{12}$ gauss), and $\omega \simeq 10 \text{ Hz}$, we find

$$\tau_2^{-1} \simeq 10^{27}. \quad (61)$$

As we will see in the next section, the value of $\alpha(\omega)$ which will give rise to saturation is about 40. Hence from equation (59) we obtain the lower limit for the saturated value of $F(\omega)$:

$$F(\omega) \sim 2 \times 10^{25}. \quad (62)$$

The brightness temperature T_B is defined as

$$F(\omega) = \frac{kT_B}{\hbar\omega} \cong 10^2 T_B \quad (\omega \simeq 10^9). \quad (63)$$

Comparing equation (62) with equation (63), we find that at saturation the lower limit of the brightness temperature is about $10^{23} \text{ }^\circ\text{K}$. However, this value of T_B is an underestimate, for equation (59) gives the condition for the onset of saturation, and when saturation fully takes place, the value of $F(\omega)\alpha(\omega)\tau_2^{-1}$ depends on the steady-state rate N_0 and other factors and may be several orders of magnitude greater than its value at the onset of saturation. A crude estimate gives a range of 10^{21} – $10^{28} \text{ }^\circ\text{K}$, and in some very unusual cases $10^{31} \text{ }^\circ\text{K}$ can also be reached.

VIII. APPLICATION TO PULSARS

In the solutions of the Boltzmann equations (51) and (54) there are two undetermined parameters, the electric field E and the displaced momentum p_m . We will now show that

these two parameters are adequately accommodated by conditions existing on the surfaces of neutron stars.

Let us assume that p_m and N_e are both independent of s so that equation (52) can be integrated. We then have

$$I(\omega, s) = I(\omega, 0) \exp [\ln (\gamma\lambda)^{-\delta s p_m^{-3}}] = I(\omega, 0)(\gamma\lambda)^{-\delta s p_m^{-3}}. \quad (64)$$

Now we need to determine $I(\omega, 0)$. It is very tempting to insert for $I(\omega, 0)$ the blackbody radiation spectrum at the temperature of the medium. But in doing so we purposely assume that our "radiation amplifier" is clean and noise-free, which is by no means the case. In fact, this radiation amplifier is a very noisy one, and it is actually the noise that this amplifier is amplifying. Therefore, we should use for $I(\omega, 0)$ the power spectrum of noise, which is Johnson noise and is proportional to a noise temperature T_N . Therefore,

$$I(\omega, 0)d\omega = I_0 T_N d\omega. \quad (65)$$

The proportionality constant I_0 can be roughly estimated by the central frequency of interest (say, $\omega = 10^9$ Hz) by equating $I(\omega, 0)$ to the blackbody radiation density at that frequency. This gives

$$I(\omega, 0) = 2kT_N \nu_m^2 c^{-2}. \quad (66)$$

Equation (64) then becomes

$$I(\omega, s) = 2kT_N \nu_m^2 c^{-2} (\gamma\lambda)^{-\delta s p_m^{-3}}. \quad (67)$$

The condition for saturation is fulfilled for a typical model at $T \sim 10^{24}$ ° K (see the previous section). With a value of $T_N \sim 10^6$ ° K, in order for saturation to take place, the value of $\delta s p_m^{-3}$ is approximately 3.5. This means that after radiation has traveled a distance s such that $\delta s p_m^{-3}$ is 3.5, the amplification of radiation does not take place exponentially as the distance. It is clear that the final amplification factor after the saturation condition is reached depends on the physical condition of individual objects, but the range of the brightness temperature is narrower than if the saturation condition were absent.

We will now show that the value of the electric field and the displaced momentum needed to have saturation are entirely reasonable. The two equations which determine the field E and the displaced momentum p_m are:

$$eE = \frac{5}{4}(2m)^2 \hbar W_0 p_m^{-4} c_1(\lambda) E_\gamma \quad (68)$$

where E_γ is the radiation energy density,

$$E_\gamma = g_\gamma \hbar^{-3} \int d^3 p_\gamma F(\omega) \hbar \omega, \quad (69)$$

and the condition for saturation that

$$\frac{I(\omega, s)}{I(\omega, 0)} \approx 10^{24} \text{ }^\circ \text{K}. \quad (70)$$

From equation (37) we find

$$F(\omega, s) = F(\omega, 0)(\gamma\lambda)^{-\eta}, \quad (71)$$

where $\eta = 2\delta s p_m^{-3}$. The saturation condition gives $\eta \sim 7$, and this fixes p_m :

$$p_m \simeq 10^{-2} m c s^{1/3}, \quad (72)$$

where s is the distance over which the laser becomes saturated. If $s = 1$ cm, then $p_m \sim 10^{-2}$. The energy associated with macroscopic streaming motion is of the order of 10 eV per particle, which is quite small.

The internal electric field required for saturation is of the order of 0.1 V cm^{-1} . This is small compared with the field just outside the star, which is of the order of 10^8 V cm^{-1} or more. However, it is well known that the electric field inside a perfect conductor must be zero, independent of the amount of current flow. In our case, although the conductivity is very high ($\sim 10^{22}$), it is not contradictory that the internal field is only of the order of 0.1 V cm^{-1} even when the field outside in vacuum is 10^8 V cm^{-1} .

IX. POLARIZATION OF RADIATION

In previous papers (Chiu and Canuto 1969*b*, *c*; Chiu, Canuto, and Fassio-Canuto 1969; Chiu 1969) we have suggested that the propagation of radiation is strongly affected by the magnetic field resulting in a beamed emission. In this section we will discuss the dielectric properties of a plasma in a magnetic field.

As the frequency of radiation is small compared with the Larmor frequency of the ions and the electrons, as well as the plasma frequency of the electrons and the ions, we can use the dielectric constant computed for a cold ionic neutral plasma in a magnetic field. It turns out that quantum-mechanical calculations (Kelly 1964) and classical calculations coincide in this limit:

The dielectric tensor is (assuming that the magnetic field is in the z -direction) (Stix 1962):

$$\epsilon_{\alpha\beta} = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix}, \quad (73)$$

where

$$R = 1 - \frac{\omega_p^2}{\omega^2} \frac{\omega}{\omega - \omega_H} - \frac{\omega_I^2}{\omega^2} \frac{\omega}{\omega + \Omega_H}, \quad (74)$$

$$L = 1 - \frac{\omega_p^2}{\omega^2} \frac{\omega}{\omega + \omega_H} - \frac{\omega_I^2}{\omega^2} \frac{\omega}{\omega - \Omega_H},$$

$$P = 1 - \left(\frac{\omega_p}{\omega}\right)^2 - \left(\frac{\omega_I}{\omega}\right)^2, \quad 2D = R - L, \quad 2S = R + L, \quad (75)$$

$$\omega_p^2 = 4\pi e^2 N_e / m, \quad \omega_I^2 = 4\pi e^2 Z N_i / m,$$

$$\omega_H = eH / mc, \quad \Omega_H = Z\omega_H m / m_i, \quad (76)$$

where ω_p and ω_I are plasma frequencies of the electron gas and the ion gas, respectively, and ω_H and Ω_H are the corresponding Larmor frequencies.

Generally speaking, the photon propagation in a doubly refractive medium such as equation (73) can be analyzed into two modes, the ordinary and the extraordinary mode of propagation. The propagation of these two modes can be easily studied for the direction $\theta = 0$ and $\theta = \frac{1}{2}\pi$, and can be studied numerically at other angles. Here we will describe the cases $\theta = 0$ and $\theta = \frac{1}{2}\pi$, respectively.

a) Along the Magnetic Field $\theta = 0$

In this direction these two modes are right-hand (R) and left-hand (L) circularly polarized, and the refractive indices for these two modes are: (o = ordinary, x = extraordinary)

$$n_x^2 = R, \quad (77)$$

$$n_o^2 = L. \quad (78)$$

The emissivity of the x - and the o -modes in the $0 \rightarrow 0$ bremsstrahlung transition is

$$\text{Ordinary mode:} \quad W^{(o)} = L^{1/2} W_0(p\omega)^{-1} C_1(\lambda) E_1^{-2}, \quad (79)$$

$$\text{Extraordinary mode:} \quad W^{(x)} = R^{1/2} W_0(p\omega)^{-1} C_1(\lambda) E_2^{-2}, \quad (80)$$

where

$$E_1^{-1} = (H/H_q)[\hbar\omega/mc^2 - H/H_q - \frac{1}{2}L(\hbar\omega/mc^2) - L^{1/2}(\hbar\omega/mc^2)(p/mc)]^{-1}, \quad (81)$$

$$E_2^{-1} = (H/H_q)[\hbar\omega/mc^2 + H/H_q + \frac{1}{2}R(\hbar\omega/mc^2) - R^{1/2}(\hbar\omega/mc^2)(p/mc)]^{-1}. \quad (82)$$

Now we will substitute into these equations quantities pertinent to a neutron star: $H = 10^{12}$ gauss, $N_e = 10^{24}$. We find $\omega_p \simeq 10^{16}$, $\omega_I \simeq 10^{14}$, $\omega_H \simeq 10^{19}$, $\Omega_H \simeq 10^{16}$. If we choose $\omega = 10^9$ Hz, we find that $E_1^{-2} = E_2^{-2} = 1$. For R and L we can expand equations (74) and (75) into power series in ω/ω_H and ω/Ω_H . For a charge-neutral plasma, for which equation (76) is valid, we find that the first nonvanishing term in the expansion gives

$$R = 1 + \frac{\omega_p^2}{\omega_H^2} + \frac{\omega_I^2}{\Omega_H^2} \simeq 1, \quad (83)$$

$$L = 1 - \frac{\omega_p^2}{\omega_H^2} - \frac{\omega_I^2}{\Omega_H^2} \simeq 1; \quad (84)$$

and hence

$$W^{(o)}(p) = W^{(x)}(p) = W_0(p\omega)^{-1} C_1(\lambda). \quad (85)$$

Therefore, these two modes have the same emissivity as that in vacuum. (This is the reason why we made no distinction between these two modes in eq. [14].) We therefore conclude that the radiation emerging from the dense plasma of the star along the magnetic field has both right- and left-hand polarization and has the same intensity. It is not known whether these two modes can combine into a linearly polarized beam or whether these two modes will propagate separately to yield an unpolarized beam.

b) Propagation Perpendicular to the Field, $\theta = \frac{1}{2}\pi$

In the perpendicular direction there is only one mode of propagation; this is the extraordinary mode. The refractive index for the ordinary mode is simply $1 < (\omega_I/\omega)^2 + (\omega_p/\omega)^2$, and propagation of this mode is forbidden. For the extraordinary mode the refractive index is:

$$n_x^2 = RL/S \approx 1, \quad (86)$$

and propagation is the same as in vacuum. This mode has 100 percent linear polarization whose direction is perpendicular to both the wave vector k and the magnetic field H . There is another mode of emission whose electric field is along the magnetic field. Our computation shows that this mode is important only in vacuum. However, since this mode cannot propagate in a dense plasma, it is not considered here.

c) Propagation along an Arbitrary Direction

As it turns out, the refractive index for the extraordinary mode is always close to unity for the field strength and plasma frequency we have quoted. The emerged radiation has an elliptical polarization which can be decomposed into a circular polarization plus a linear polarization. The polarization of the radiation from bremsstrahlung process that can emerge from the medium can be either circularly or linearly polarized, depending on the emissivity associated with these two polarization states. However, from simple arguments it seems that linear polarization is favored over circular polarization.

Figure 3 shows the directions of polarization of the emerging radiation. It is clear that if we look into the direction of the magnetic field, the radiation will appear unpolarized, assuming that these two modes (o and x) propagate and become amplified independently of each other. However, if we look at the medium at an oblique angle, the radiation will appear partially linearly polarized, and the degree and inclination of polarization with respect to a fixed plane of rotation will depend on the relative orientation of the emitting surface with respect to the observer. This is illustrated in Figure 4. Thus, in general, we expect that pulsar radiation will be linearly polarized and the polarization strength and orientation will change through the cycle. Observation shows that in some cases pulsar radiation is found to be polarized and the orientation and the degree of polarization change with the phase of the pulse (Radhakrishnan and Cooke 1969). We are now in the process of extending our computation of the emissivity of radiation at an arbitrary angle so that a comparison of our theory with observation can be made.

X. DISCUSSIONS OF THE PRESENT MODEL AND FUTURE ASPECTS

The first obvious direction of improvement of our theory is to use an improved electron distribution function. In the δ -function approximation there is an inconsistency, but paradoxically, this inconsistency is necessary in order to yield a correct result. Throughout we have used a value of ϵ which is different from the displaced energy $\epsilon_m = p_m^2/2m$. The reason is as follows: If we use a better electron distribution function, p_m will become the average energy of the electron, so that this "inconsistency" is necessary in order to upset the "ill" effects of the δ -function. It is then easy to see that the use of a better electron distribution function will not alter the resulting spectrum, as the use of a better distribution function only alters quantities now expressed as functions of p_m , replacing those quantities by better averages. Because of cancellation between quantities in the $+p$ and in the $-p$ directions (cf. eqs. [29], [30], and [36]), the electric field required might be larger.

The second improvement is to include other processes (such as Compton scattering) and solve equation (54) in conjunction with the Boltzmann equation to obtain a better value for the saturated brightness temperature. Nonlinear effects will take place, and pulsed mode of emission may take place.

These improvements, however, are only refinements to our theory. The essence of the theory is already contained in this very crude model.

This model, crude as it is, can explain semiquantitatively the following features of pulsar observation:

1. That the brightness temperature of radiation from pulsars is in the range 10^{20}° – 10^{30}° K and is sensitive to the pulsar periods.
2. That the radiation is emitted at the polar region.
3. That the radiation has a component showing linear polarization and the degree and angular inclination of polarization change with the phase of the pulse.

Many other features of pulsar emission remain to be explained: the existence of the precursor in the pulses, the existence of one or more periods, the behavior of polarization during the pulse, the broadening of pulses at low frequencies, the shape of the pulse, and so on. But we believe that these features may well depend on the detailed structure of the neutron star and the surrounding medium. The fundamental question posed by the pulsar is why a neutron star should radiate predominantly in the radio region with the high brightness temperature of 10^{24}° K or more when one expects that the neutron star should radiate in the X-ray band, according to the blackbody radiation law. The highest brightness temperature of other radio sources is less than 10^9° K, and this difference in the brightness temperature between an ordinary radio source and a pulsar is the most convincing indication that the physics of pulsar radiation must be drastically different from that of ordinary radio sources. (While in an ordinary radio source a brightness temperature of 10^9° K can be achieved by synchrotron radiation by par-

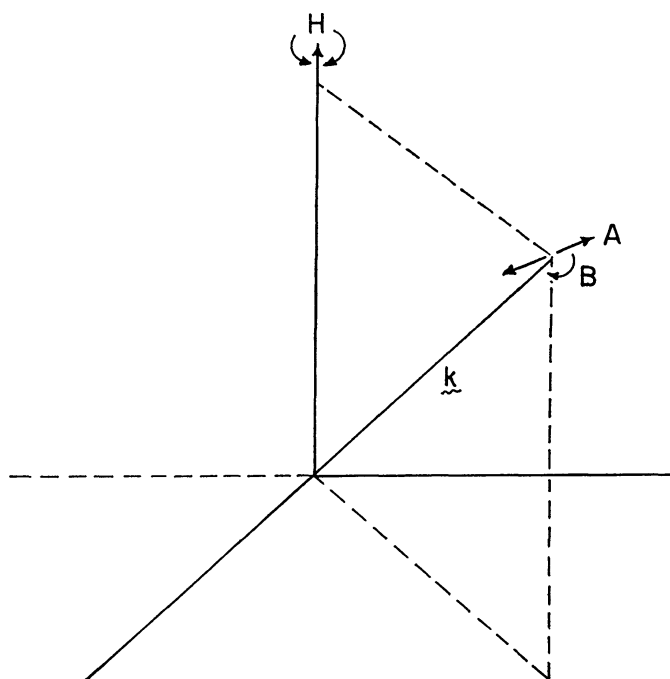


FIG. 3.—Polarization of the bremsstrahlung radiation in a strongly magnetized medium. K = propagation vector, H = magnetic field. A is the linearly polarized component perpendicular to the field; B is the circularly polarized component of the extraordinary component.

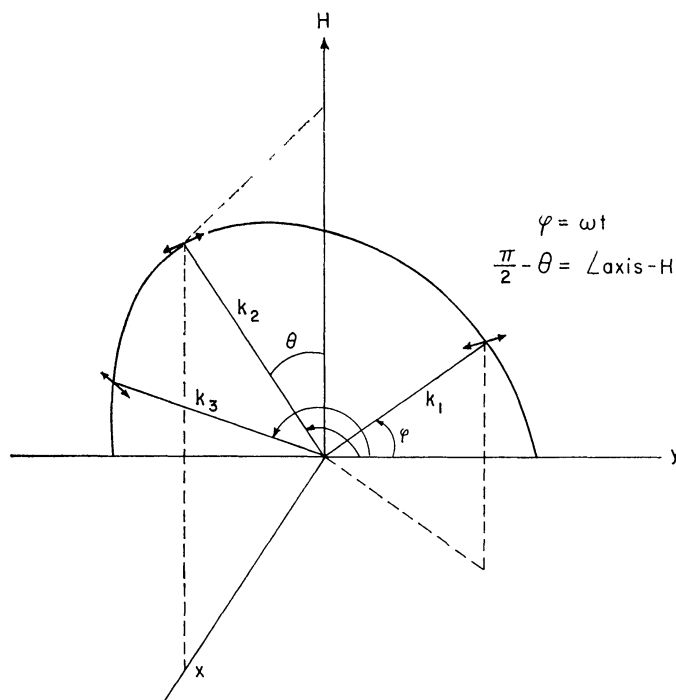


FIG. 4.—Relative orientation of polarization in the observer's plane for a pulsar whose rotational axis is perpendicular to the line of sight and whose magnetic field is inclined at an angle $\frac{1}{2}\pi - \theta$ with respect to the rotational axis.

ticles with energies of the order of BeV or greater, it is difficult, if not impossible, to explain the pulsar radiation by even the most articulate conditions such as plasma coherence.) Indeed, as we have shown, an electron gas in a magnetic field of 10^{12} gauss should be expected to emit continuum laser (C-maser) radiation of a brightness temperature corresponding to that from pulsars if coherent relative motions of electrons and ions exist.

In conclusion, we believe that we have arrived at a reasonable and quantitative explanation of the radiation mechanism of pulsars, without assuming a priori the artificial existence of energetic particles at great distances from the neutron star. The difficulty associated with coherent emissions from a plasma, that is, the bandwidth of emission is $\sim \frac{1}{2}\lambda$, is also avoided.

We would like to thank Dr. Stephen P. Maran for his interest in our theory throughout its development, and for many discussions on the observational aspects of pulsars. Dr. D. C. Kelly's critical reading of the manuscript is deeply appreciated. We would like to thank Drs. Malvin A. Ruderman and E. E. Salpeter for numerous discussions. One of us (V. C.) acknowledges an NAS-NRC Associateship, and wishes to thank Dr. Robert Jastrow for the hospitality of the Institute for Space Studies.

REFERENCES

- Böhm-Vitense, E. 1969, *Ap. J. (Letters)*, **156**, L131.
 Bradt, H., Rappaport, S., Mayer, W., Nather, R. E., Warner, B., MacFarlane, M., and Kristian, J. 1969, *Nature*, **222**, 728.
 Canuto, V., and Chiu, H. Y. 1968, *Phys. Rev.*, **173**, 1210, 1220, 1229.
 ———. 1970, *Phys. Rev.*, **A2**, 518.
 Chiu, H. Y. 1968, *Stellar Physics* (Waltham, Mass.: Blaisdell Publishing Co.).
 ———. 1970, in preparation.
 Chiu, H. Y., and Canuto, V. 1969a, *Nature*, **225**, 1230.
 ———. 1969b, *Phys. Rev. Letters*, **22**, 415.
 ———. 1969c, *Nature*, **223**, 1113.
 ———. 1970, in preparation.
 Chiu, H. Y., Canuto, V., and Fassio-Canuto, L. 1969, *Nature*, **221**, 529.
 Chiu, H. Y., and Fassio-Canuto, L. 1969, *Phys. Rev.*, **185**, 1614.
 Drake, F., and Craft, H. D. 1968, *Nature*, **220**, 231.
 Dreicer, H. 1959, *Phys. Rev.*, **115**, 238.
 ———. 1960, *ibid.*, **117**, 329.
 Erber, T. 1966, *Rev. Mod. Phys.*, **38**, 628.
 Gold, T. 1969, *Nature*, **221**, 25.
 Goldman, R., and Oster, L. 1964, *Phys. Rev.*, **136A**, 602.
 Goldreich, P., and Julian, W. H. 1969, *Ap. J.*, **157**, 869.
 Kelly, D. 1964, *Phys. Rev.*, **A641**, 134.
 Maran, S. P., and Cameron, A. G. W. 1969, *Earth and Extraterr. Sci.*, **1**, 3.
 Ostriker, J. P., and Gunn, J. E. 1969, *Ap. J.*, **157**, 1395.
 Pacini, F. 1968, *Nature*, **219**, 145.
 Radhakrishnan, V., and Cooke, D. J. 1969, *Ap. Letters*, **3**, 225.
 Reichley, P. E., and Downs, G. S. 1969, *Nature*, **222**, 229.
 Roberts, J. A. 1969, *Nature*, **222**, 862.
 Ruderman, M. A. 1968, *Nature*, **218**, 1128.
 ———. 1969, *ibid.*, **223**, 597.
 Simon, M., and Strange, D. L. P. 1969, *Nature*, **224**, 49.
 Stix, T. H. 1962, *The Theory of Plasma Waves* (New York: McGraw-Hill Book Co.).
 Troup, G. 1963, *Masers and Lasers* (London: Methuen).
 Wampler, E. J., Scargle, J. D., and Miller, J. S. 1969, *Ap. J.*, **157**, 21.
 Wilkinson, D. T. 1969, private communication.